The theoretical curve of Cheng's shock-layer theory presented in Fig. 3 has been deduced from the heat transfer coefficient C_H of Ref. 5 (refer to Fig. 6 of Ref. 5) for $Pr = 0.71$, $\epsilon = 0.13$, and $T_w/T_0 \rightarrow 0$. In converting K^2 to Re_F, Fig. 1 has been used. The small differences in T_w/T_0 and ϵ between the experimental and theoretical data are not believed to affect significantly the quantities presented in the correlation. The data presented for comparison encompass both the regime where vorticity interaction dominates ($Re_F \gtrsim 500$) and the regime where the transport effects immediately behind the shock are important, $Re_F \gtrsim 500$. Although the experimental data appear to rise somewhat above Cheng's values, especially in the higher Reynolds number range, the comparison in Fig. 3 indicates a general agreement between experiment and the shock-layer theory to a degree consistent with the approximation of the theory. Also included in Fig. 3 are two theoretical curves of Ferri et al. (4,7).

A somewhat similar comparison has been given recently by Ferri, Zakkay, and Ting (9), covering mainly the higher Reynolds number regime ($Re_F \gtrsim 500$). There, the agreement between the heat transfer measurement and the prediction based on the shock-layer theory of Ref. 5 seems to be even better.[7]

References

1 Hayes, W. D. and Probstein, R., *Hypersonic Flow Theory* (Academic Press, New York, 1959), p. 370.

2 Rott, N. and Leonard, M., "Vorticity effect on stagnation-point flow of a viscous incompressible fluid," J. Aerospace Sci. **26**, 542 (1959).

3 Van Dyke, M., "Second-order compressible boundary-layer theory with application to blunt bodies in hypersonic flow," Stanford Univ. Rept. SUDAER 112, Air Force Office Sci. Research TN-61-1270 (July 1961).

4 Ferri, A., Zakkay, V., and Ting, L., "Blunt body heat transfer at hypersonic speed and low Reynolds number," J. Aerospace Sci. **28**, 962 (1961).

5 Cheng, H. K., "Hypersonic shock-layer theory of the stagnation region at low Reynolds number," *Proceedings of the 1961 Heat Transfer and Fluid Mechanics Institute* (Stanford University Press, Stanford, Calif., 1961), p. 161.

6 Cheng, H. K. and Chang, A. L., "Hypersonic shock layer at low Reynolds number—the yawed cylinder," Cornell Aeronaut. Lab. Rept. AF-1270-A-4 (August 1962).

7 Ferri, A. and Zakkay, V., "Measurements of stagnation point heat transfer at low Reynolds numbers," J. Aerospace Sci. **29**, 847 (1962).

8 Eckert, E. R. G., "Engineering relations for friction and heat transfer to surface in high velocity flow," J. Aeronaut. Sci. **22**, 585 (1955).

9 Ferri, A., Zakkay, V., and Ting, L., "On blunt-body heat transfer at hypersonic speed and low Reynolds numbers," J. Aerospace Sci. **29**, 882 (1962).

[7] The apparent improvement in the agreement between experiment and the shock-layer theory reported in Ref. 9 results primarily from using the Fay and Riddell formula for Q_{BL} instead of the value obtained from the shock-layer analysis. Since the theoretical values of heat transfer rate are given in Ref. 5 as a leading approximation for small ϵ, the quantity $(Q - Q_{BL})/Q_{BL}$ based on this theory is valid (to the leading approximation) only if Q_{BL} is also taken from the leading approximation for small ϵ. Using Fay and Riddell's formula for Q_{BL} would result in misleading values of $(Q - Q_{BL})/Q_{BL}$.

Optimum Thrust Programming of Electrically Powered Rocket Vehicles for Earth Escape

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Nomenclature

a = thrust per unit mass of vehicle
m = mass of propellant expended at time t
M_0 = initial mass of vehicle plus propellant
P = jet power
r = radius measured from center of earth
s = distance measured along flight path
U = gravitational potential, $U = \mu/r$
V = velocity of vehicle
α = $(d^2U/ds^2)^{1/2}$
μ = constant of gravitational field
ϕ = flight path angle measured from local horizontal

Subscripts

0 = initial value at $t = 0$
T = final value at $t = T$

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OPTIMUM thrust programming of electrically powered rockets under the conditions of constant jet power and tangential thrust was discussed in Ref. 1. Assuming the gradient along the flight path of the gravitational force per unit mass to be constant, thrust programs were derived yielding minimum propellant utilization for the two particular cases of specified change in velocity with range arbitrary and specified range with change of velocity arbitrary. A practical problem that is actually a combination of these two cases is that of escape from a satellite orbit with a minimum expenditure of propellant. Here, neither the change in velocity nor the range is specified, but a functional relationship exists between these two quantities.

If the nomenclature of Ref. 1 is used, the equations of motion are

$$\dot{V} = a + (dU/ds) \quad [1]$$

$$\dot{s} = V \quad [2]$$

where U, the gravitational potential, is assumed to be a function of s only. The propellant mass is uniquely determined by the function

$$\psi = 2P/(M_0 - m) \quad [3]$$

where

$$\dot{\psi} = a^2 \quad [4]$$

At the specified time of burnout, T, the relationship between the velocity and the distance is given by

$$(V_T^2/2) - U_T = 0 \quad [5]$$

Upon applying the formal methods of the calculus of variations to the problem of determining $a(t)$ for which ψ_T is stationary under the restrictions imposed by Eqs. [1, 2, and 4], the differential equation

$$\ddot{a} = (d^2U/ds^2)a \quad [6]$$

is obtained, together with the condition

$$-a_T\delta V_T + \dot{a}_T\delta s_T = 0 \quad [7]$$

at time T (1).[2] From Eq. [5]

$$V_T\delta V_T - (dU/ds)_T\delta s_T = 0 \quad [8]$$

The combination of Eqs. [7] and [8] yields

$$\frac{\dot{a}_T}{a_T} = \frac{(dU/ds)_T}{V_T} \quad [9]$$

In order to obtain analytical solutions of Eq. [6], it is convenient to assume that d^2U/ds^2, the gradient of the tangential component of gravitational force per unit mass, is constant. The solution of Eq. [6] for which $a = a_0$ at $t = 0$ and Eq. [9] is satisfied at $t = T$ is then

$$\frac{a}{a_0} = \frac{\cosh\alpha T\left(1 - \frac{t}{T}\right) - K\sinh\alpha T\left(1 - \frac{t}{T}\right)}{\cosh\alpha T - K\sinh\alpha T} \quad [10]$$

[2] Numbers in parentheses indicate References at end of paper.

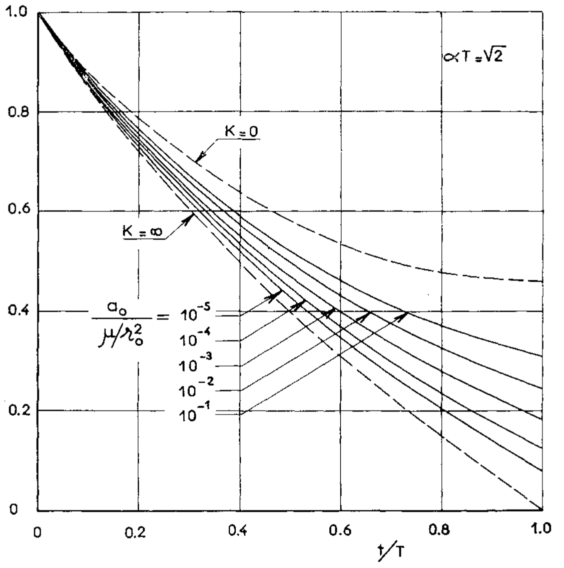

Fig. 1 Optimum thrust acceleration programs for escape from satellite orbit

where

$$K = \frac{1}{\alpha}\frac{(dU/ds)_T}{V_T} \qquad [11]$$

For the case where V_T is specified and s_T is arbitrary, $\dot{a}_T = 0$ (1) and, as indicated by Eqs. [9] and [11], $K = 0$; with s_T specified and V_T arbitrary, on the other hand, $a_T = 0$ (1) and $K = \infty$. For the problem of earth-escape, representative values of K can be obtained by assuming that, as a first approximation, a is constant and equal to a_0 along the entire trajectory. Under this assumption the low thrust rocket trajectory data of Ref. 2 can be used directly, giving $\phi_T = 39°$ at the instant when the vehicle velocity is equal to the local parabolic velocity. At burnout, therefore,

$$(dU/ds)_T \cong -(\mu/r_T^2) \sin 39° \qquad [12]$$

whereas the burnout velocity is given by

$$V_T = (2\mu/r_T)^{1/2} \qquad [13]$$

The initial tangential component of gravitational force per unit mass is $2a_0$, whereas at burnout this component is approximately a_0 (2). The range Δs is approximately $V_0\, T/2$, and V_0 is simply $(\varphi/r_0)^{1/2}$. From the definition of α, therefore,

$$\alpha^2 \cong \frac{a_0}{\Delta s} \cong \frac{2a_0}{(\mu/r_0)^{1/2}T} \qquad [14]$$

Finally, from Ref. 2

$$T \cong V_0/a_0 \cong (1/a_0)(\mu/r_0)^{1/2} \qquad [15]$$

and

$$\frac{r_0}{r_T} \cong \left[\frac{a_0}{(\mu/r_0^2)}\right]^{1/2} \qquad [16]$$

The combination of Eqs. [11–16] results in

$$K \cong -\frac{0.31}{\left(\dfrac{a_0}{\mu/r_0^2}\right)^{1/4}} \qquad [17]$$

where the quantity in parentheses in the denominator of the right-hand side represents the ratio of thrust to weight in the initial satellite orbit. The value of αT is obtained by combining Eqs. [14] and [15], giving $\alpha T = (2)^{1/2}$ regardless of the initial thrust-to-weight ratio.[3]

Eq. [10] has been plotted in Fig. 1 for $\alpha T = (2)^{1/2}$ and for values of $a_0 r_0^2/\mu$ varying from 10^{-5} to 10^{-1}. The corresponding values of K were obtained from Eq. [17]. Also shown in Fig. 1 are curves of $a(t)$ for $K = 0$ and for $K = \infty$. It can be seen that, with initial thrust-to-weight ratios of 10^{-5} and 10^{-4}, thrust programs for escape with minimum propellant utilization approach the optimum thrust program for specified range and arbitrary final velocity ($K = \infty$). With larger initial thrust-to-weight ratios, however, optimum thrust programs for escape tend toward the optimum curve for specified final velocity and arbitrary range ($K = 0$). Regardless of the initial thrust-to-weight ratio, the departure of the optimum earth-escape thrust program from that of constant thrust per unit mass is considerable.

References

1 Faulders, C. R., "Optimum thrust programming of electrically powered rocket vehicles in a gravitational field," ARS J. **30**, 954–960 (1960).

2 Perkins, F. M., "Flight mechanics of low thrust space craft," *Vistas in Astronautics* (Pergamon Press Inc., New York, 1959), Vol. II; J. Aerospace Sci. **26**, 291–297 (1959).

[3] This value of αT represents an improvement over the values reported in Table 3 of Ref. 1.

Electrical Discharge Across a Supersonic Jet of Plasma in Transverse Magnetic Field[1]

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Observations have been made of the discharge between two electrodes placed across a supersonic stream of ionized gas. A magnetic field transverse to both the flow direction and the electrode axes was applied. In this note, visual observations of the behavior of the discharge with and without magnetic fields will be described, and qualitative explanations of the observed phenomena will be proposed.

THE experiments described were performed with water-cooled copper electrodes in contact with the cooler outer sheath of a free jet of argon plasma produced by a commercial arcjet (Plasmadyne M-4). Argon gas at a flow rate of 1.36×10^{-3} kg/sec was heated to an average stagnation enthalpy of the order of 8×10^6 j/kg. The plasma entered the region between the electrodes at a velocity of approximately 3×10^4 m/sec and a Mach number of approximately 2.5. The electrode cross-sectional area was 6.45×10^{-4} m², and the electrode gap (i.e., the separation between the two electrode surfaces) was 3.8×10^{-2} m. Further details of the apparatus are given elsewhere (1–3).[4]

The contact of the electrodes with the plasma stream at the operating pressures of from 2 to 4 mm Hg generated weak, oblique shock patterns that were faintly but distinctly visible

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[4] Numbers in parentheses indicate References at end of paper.